SOLUTION SET VII

EXERCISE VII.1 : INCREASE OF COSϕ OF A RL LOAD

A. $P = U \cdot I \cdot \cos\phi \quad \Rightarrow \quad I = \dfrac{P}{U \cdot \cos\varphi} = \dfrac{45,5 \cdot 10^3}{380 \cdot 0,6} = \underline{199,56\ A}$

$\phi = \arccos(0,6) = 53,13°$

$\underline{I} = \underline{199,56 \cdot e^{-j\,53,13°}\ A}$

$\underline{Z} = Z \cdot \cos\phi + j\,Z \cdot \sin\phi = R + j\omega L \quad \Rightarrow \quad R = Z \cdot \cos\phi \quad ; \quad \omega L = Z \cdot \sin\phi$

$Z = \dfrac{U}{I} = \dfrac{380}{199,56} = 1,9\ \Omega$

$R = 1,9 \cdot 0,6 = \underline{1,14\ \Omega}$ and $L = \dfrac{1,9 \cdot 0,8}{100 \cdot \pi} = \underline{4,84\ mH}$

B. $\underline{Z}_M = R + j\,\omega L \quad \Rightarrow \quad \underline{Y}_M = \dfrac{1}{R + j\,\omega L} = \dfrac{R - j\,\omega L}{R^2 + (\omega L)^2}$ and $\underline{Y}_C = j\,\omega C$

$$\underline{Y} = \underline{Y}_M + \underline{Y}_C = \frac{R - j\,\omega L}{R^2 + (\omega L)^2} + j\omega C = \frac{R}{R^2 + (\omega L)^2} - j\,\omega\left(\frac{L}{R^2 + (\omega L)^2} - C\right)$$

$\phi' = \arccos(0,95) = \pm 18,19°$ (2 possible solutions)

$\phi' = \arg(\underline{Z}) = -\arg(\underline{Y})$

$$\tan\phi' = \frac{\dfrac{\omega L}{R^2 + (\omega L)^2} - \omega C}{\dfrac{R}{R^2 + (\omega L)^2}} = \frac{\omega L - \omega C\left[R^2 + (\omega L)^2\right]}{R}$$

$$C = \frac{\omega L - R\tan\phi'}{\omega\left[R^2 + (\omega L)^2\right]} = \begin{cases} \underline{1008\ \mu F} & (\text{pour } \phi' = +18,19°) \\ \underline{1667\ \mu F} & (\text{pour } \phi' = -18,19°) \end{cases}$$

C. $\underline{Y} = \underline{(0,316 - j \cdot 0,104)\ S} = \underline{0,3327 \cdot e^{-j\cdot 18,19°}\ S}$

$\underline{I}' = \underline{Y} \cdot \underline{U} = \underline{126,4 \cdot e^{-j\,18,19°}\ A}$

D. $P_{pertes} = R_L \cdot I^2 = \underline{1,99\ kW}$

$P'_{pertes} = R_L \cdot I'^2 = \underline{0,79\ kW}$

E. Decrease of losses in lines

EXERCISE VII.2 : POWER IN ALTERNATIVE REGIME

If $\underline{Z}_1$ is capacitive :

$\underline{Z}_1 = R_1 - j\dfrac{1}{\omega C_1}$ where C_1 is a capacitor supposed to be in series with R_1

$$\Rightarrow \quad R_1 = \frac{U}{I_1}\cos\phi = \underline{8\ \Omega} \quad \text{and :} \quad X_1 = \frac{U}{I_1}\sin\phi = \frac{1}{C_1\omega} \quad \Rightarrow \quad C_1 = \frac{I_1}{U}\cdot\frac{1}{\omega\sin\phi} = \underline{530{,}5\ \mu F}$$

When adding a capacitor C in series with $\underline{Z}_1$, C_1 is replaced by $C_1' = \left(\dfrac{1}{C} + \dfrac{1}{C_1}\right)^{-1}$, X_1 becomes X_1', ϕ becomes ϕ' and I_1 becomes I_2 :

$$R_1 = \frac{U}{I_2}\cos\phi' = 8\ \Omega \quad \text{and :} \quad X_1' = \frac{U}{I_2}\sin\phi' = \frac{1}{C_1'\omega}$$

$$\Rightarrow \quad I_2 = \frac{U\cos\phi'}{R_1} = \underline{7{,}5\ A} \quad \text{and :} \quad R_1 tg\phi' = \frac{1}{\omega}\cdot\left(\frac{1}{C} + \frac{1}{C_1}\right) \Rightarrow C = \frac{1}{\omega R_1 tg\phi' - 1/C_1} = \underline{682{,}1\ \mu F}$$

If $\underline{Z}_1$ is inductive :

$\underline{Z}_1 = R_1 + j\,\omega L_1$ where L_1 is an inductor supposed to be in series with R_1

$$\Rightarrow \quad R_1 = \frac{U}{I}\cos\varphi = \underline{8\ \Omega} \quad \text{and :} \quad X_1 = \frac{U}{I}\sin\varphi = L_1\omega \quad \Rightarrow \quad L_1 = \frac{U}{I}\cdot\frac{\sin\varphi}{\omega} = \underline{19{,}1\ mH}$$

When adding a capacitor, the impedance becomes $\underline{Z}_2$:

$$\underline{Z}_2 = R_1 + j\left(L_1\omega - \frac{1}{\omega C}\right)$$

$$\Rightarrow \quad R_1 = \frac{U}{I_2}\cos\phi' = 8\ \Omega \quad \text{and :} \quad X_1' = \frac{U}{I_2}\sin\phi' = L_1\omega - \frac{1}{C\omega}$$

$$\Rightarrow \quad I_2 = \frac{U\cos\phi'}{R_1} = \underline{7{,}5\ A} \quad \text{and :} \quad R_1 tg\phi' = L_1\omega - \frac{1}{C\omega}$$

Note that if we add a <u>small</u> capacitance, the phase shift decreases, so cosϕ increases, which does not correspond to the problem given. To obtain a reduction of cosϕ, it is necessary to add a capacitor <u>large</u> enough so that the total impedance becomes capacitive. In these conditions, ϕ' is negative :

$$tg\phi' = -\,tg[\arccos(\cos\phi')]$$

$$\Rightarrow \quad -R_1 tg(\arccos(\cos\phi')) = L_1\omega - \frac{1}{C\omega} \quad \Rightarrow \quad C = \frac{1}{\omega\cdot\left[L_1\omega + R_1 tg[\arccos(\cos\phi')]\right]} = \underline{191\ \mu F}$$

EXERCISE VII.3 : ACTIVE AND REACTIVE POWER

A. The phase shift $\varphi = -\frac{\pi}{3} - \left(-\frac{\pi}{18}\right) = -\frac{5\pi}{18} = \underline{-50°} < 0$: current ahead of the voltage

$\Rightarrow$ the load is capacitive.

B. We have rms values: $U = \hat{U}/\sqrt{2}$; $I = \hat{I}/\sqrt{2}$.

Apparent power : $S = U \cdot I = ½\, \hat{U} \cdot \hat{I} = \underline{120\ \text{VA}}$

Active power : $P = U \cdot I \cos\varphi = \underline{77{,}1\ \text{W}}$

Reactive power : $Q = U \cdot I \sin\varphi = \underline{91{,}9\ \text{var}}$

C. The resistance $R = \frac{P}{I^2} = \frac{2P}{\hat{I}^2} = \underline{1{,}1\ \Omega}$ and the reactance $X = \frac{Q}{I^2} = \frac{2Q}{\hat{I}^2} = \underline{-1{,}28\ \Omega}$